

On Anyonic Variables and Fractional Supersymmetry

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A modified definition for anyonic variables is given. They are used to construct a representation for superspace of fractional supersymmetry in two dimensions. An extension to higher dimension is presented.

Introduction

The phenomena of fractional Hall effect [1, 2] and anyon superconductivity [3] have increased the interest in fractional symmetries. The attempts to apply this approach o supersymmetry [4] has been hampered by the lack of representations of variables which generalize Grassmann variables. Recently such variables have been defined [5, 6]. They are called anyonic variables.

In this work they will be slightly modified. It will be shown that they can be used as superspace variables for fractional supersymmetry (FSUSY) in $d = 2$. A nonrelativistic generalization to higher dimensions is discussed.

1. Anyonic Variables

The variables $\theta_1, \theta_2, \dots$ are called anyonic variables of type q if

$$\theta_k \theta_j = q_{kj} \theta_j \theta_k, \quad (1)$$

$$q_{kj} = \exp[iqS(k-j)],$$

$$S(k-j) = \begin{cases} 1 & \text{if } k > j, \\ -1 & \text{if } k < j, \\ 0 & \text{if } k = j. \end{cases} \quad (2)$$

Since $S(k-j) + S(j-k) = 0$, this definition is consistent. In the following only the case $q = 2\pi/n$, $n = 1, 2, 3, \dots$ will be considered. In the previous work [5, 6] only the case $q = \pi/n$ has been studied. The case $n = 1$ corresponds to community variables. It will be

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shown that $n = 2$ anyonic variables are the known Grassmann variables.

The conjugate variables $\bar{\theta}_1, \bar{\theta}_2, \dots$ satisfy the relation that, if $\eta\theta = \exp(iq)\theta\eta$, then $\eta\bar{\theta} = \exp(-iq)\bar{\theta}\eta$ and $\theta\bar{\theta} = \bar{\theta}\theta$.

Differentiation and integration are defined by analogy with Grassmann variables [7]:

$$\begin{aligned} \partial/\partial\theta_j(1) &= 0, \quad \partial/\partial\theta_j\theta_k f = q_{jk}\theta_k \partial f/\partial\theta_j \quad \text{if } k \neq j, \\ \partial/\partial\theta_j(\theta_j f) &= f + \exp(-iq)\theta_j \partial f/\partial\theta_j, \end{aligned} \quad (3)$$

consequently

$$\partial(\theta_k)^p/\partial\theta_j = \delta_{kj}(\theta_k)^{p-1} \cdot (1 - \exp(2i\pi p/n))/(1 - \exp(2i\pi/n)),$$

which vanishes identically for $p = n$. Therefore the following condition is imposed on $q = 2\pi/n$, $n > 1$ anyonic variables:

$$(\theta_k)^n = 0, \quad k = 1, 2, 3, \dots \quad (4)$$

This shows that $n = 2$ anyonic variables are identical to Grassmann ones and that anyonic variables generalize Grassmann ones to fractional ($n > 2$) cases. This generalization is not expected to be unique.

Using (4), anyonic integration is defined for $n > 1$ by

$$\begin{aligned} \int (\theta_j)^p d\theta_k &= 0 \quad \forall j, k \quad \text{if } p < n-1, \\ \int (\theta_j)^{n-1} d\theta_k &= \delta_{jk}. \end{aligned} \quad (5)$$

This definition guarantees translation invariance. For $n = 2$, Berezin formulas for Grassmann variables are regained.

2. Fractional Supersymmetry (FSUSY)

In this work it is claimed that $2\pi/n$ anyonic variables can be used to formulate a superspace-like construction for FSUSY. For $n = 2$ ordinary SUSY is regained. The case $d = 2$ will be discussed first. The generator Q for $d = 2$ FSUSY satisfies [4]

$$Q^n = P, \quad (6)$$

where P is the translation generator. The fractional superspace representation can be given, using a $2\pi/n$ anyonic variable θ , by

$$\begin{aligned} P &= C_q \partial/\partial x, \quad C_q = \left[\prod_{j=0}^{n-2} (1 + q + q^2 + \dots + q^j) \right], \\ Q &= (C_q)^{(1/n)} D, \quad D = \partial/\partial\theta + (\theta)^{n-1} \partial/\partial x, \end{aligned} \quad (7)$$



where $q = \exp(2\pi i/n)$. It is straightforward to check (7) by taking into account both (4) and the relation

$$\sum_{k=1}^n q^k = 0.$$

Having shown that anyonic variables can be used to construct a superspace-like formulation for $d = 2$ FSUSY, we will attempt a generalization to higher dimensions. In $(2 + 1)$ dimensions one can use two $2\pi/n$ anyonic variables θ_1, θ_2 and define

$$\begin{aligned} P_1 &= C_q \partial/\partial x, & P_2 &= C_q \partial/\partial y, \\ D_j &= \partial/\partial \theta_j + (\theta_j)^{n-1} \partial/\partial x_j, & x_1 &= x, x_2 = y, \\ Q_j &= (C_q)^{1/n} D_j. \end{aligned} \quad (8)$$

It can be shown that

$$(Q_j)^3 = P_j, j = 1, 2 \quad \text{and} \quad D_1 D_2 - q D_2 D_1 = 0. \quad (9)$$

Generalization to $d > 3$ is straightforward.

In order to get a relativistic formulation for (8) and (9), a generalization for the Dirac equation to the anyonic case should be found. This is a task for the future.

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